SAMPLE of Diploma Prep Materials - Math 30-1

Function Operations / Transformations

I

Course weighting: **19%** Approx. Number of Exam Questions: **8**

1.1 Operations on Functions

i – Domain of a Function

For domain of a function we do not need to consider the graph.

Ask: **Are there any restrictions?**

That is, values of x that would result in ….

❶ Dividing by 0

Example 1:

$$f(x) = \frac{(x-1)(x+2)}{(x-1)(x-2)}$$

Domain of $f(x)$

❷ Taking the square root of a negative

Example 2:

$$g(x) = \sqrt{4-2x}$$

Domain of $g(x)$

❸ Taking the LOG of 0 or a negative

Example 3:

$$h(x) = log_2(2x-3)$$

Domain of $h(x)$

Try these…

State the domain of each function *Answers are on the top of the next page*

1. $y = \frac{2x-5}{4x+3}$

2. $p(x) = 2x^2 - 3x$

3. $y = \frac{x(x-2)}{4x^2+x}$

4. $g(x) = \sqrt{4x+1}$

5. $y = log_2(x^2)$

6. $y = tan(\theta)$

7. $y = log_2(3-x)$

8. $y = \frac{2x-5}{4x^2-3}$

Domain answers from previous page

1. $x \neq -3/4$ 2. $x \in \mathbb{R}$ 3. $x \neq 0, -1/4$ 4. $x \geq -1/4$ 5. $x \neq 0$ 6. $\theta \neq \pi/2 + n\pi\,; n \in I$ 7. $x < 3$ 8. $x \neq \pm\sqrt{3}/2$

ii – Operations on Functions

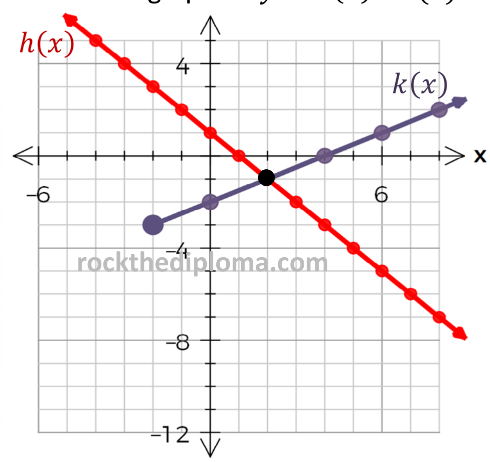
Functions (defined by equations, graphs, charts, etc) can be **added**, **subtracted**, **multiplied**, or **divided**. (Combined)

Answers for examples 1, 3, and 4 are on the top of the next page

Example 1: Given $f(x) = x^2 - 4$ and $g(x) = x - 2$, determine a simplified expression for

(a) $(f - g)(x)$ **(b)** $\left(\frac{f}{g}\right)(x)$

Example 2: Given the graphs of $h(x)$ and $k(x)$ below, sketch the graph of $y = h(x) * k(x)$



The **domain** of combined functions is like being a good host cooking a meal for two friends:

- One can't have gluten
- The other can't have meat

*So your meal must honor **both restrictions** – and not have gluten and not have meat!*

Example 3: State the domain for $y = h(x) * k(x)$, from example 2.

However when **dividing**, we have an additional restriction as the bottom function (denominator) can't be zero.

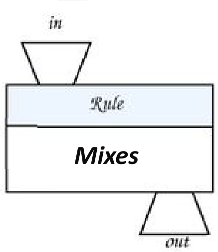
Example 4: Given $f(x) = x^2 - 4$ and $g(x) = x - 2$, from example 1, state the domain of:

(a) $(f - g)(x)$ **(b)** $\left(\frac{f}{g}\right)(x)$ **(c)** $\left(\frac{g}{f}\right)(x)$

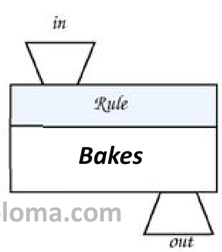
iii – Composition of Functions

Suppose with have two functions in a kitchen.

$f(x)$:



$g(x)$:



So to prepare a cake, we'd first input the **raw ingredients** in the mix function, that is, $f(x)$, to the get output "**mixed ingredients**"

We'd then take that output and input them (**mixed ingredients**) into the bake function, that is $g(x)$, to the get output "a **baked cake**"!

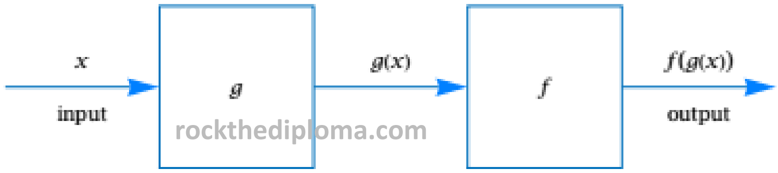
This entire, two-step operation can be expressed as $g(f(x))$ or alternatively $g \circ f(x)$

Note, this is clearly different than $f(g(x))$!

Example answers from previous page

Ex 1 (a) $x^2 - x - 2$ (b) $x + 2\,; x \neq 2$ *Ex 3* $x \geq -2$ *Ex 4* (a) $x \in \mathbb{R}$ (b) $x \neq 2$ (c) $x \neq \pm 2$

The composition $\boldsymbol{f} \circ \boldsymbol{g}$ as the combination of two machines.



Here $g(x)$ is the output of the $\boldsymbol{g}$ machine and $g(x)$ is also the input of the $\boldsymbol{f}$ machine.

*Answers are on the bottom of **this page***

Example 1: Given $g(x) = 2x - 2$ and $h(x) = x^2$ determine

(a) $g(h(3))$ **(b)** $h \circ g(0)$

Example 2: Given $f(x) = \sqrt{x+1}$, $g(x) = 2x - 2$ and $h(x) = x^2$ determine simplified expressions for

(a) $g(g(x))$ **(b)** $h \circ f \circ g(x)$

To find the **domain** of a composite function, such as $f \circ g(x)$, first find and simplify the composite function, and then consider any restrictions. *(Dividing by zero, square rooting negatives, etc)*

Example 3: Given $f(x) = \sqrt{x+1}$, $g(x) = 2x - 2$ and $h(x) = x^2$ determine the domain of

(a) $g(f(x))$ **(b)** $f \circ g(x)$ **(c)** $h \circ f \circ g(x)$

Example answers from this page

Ex 1 (a) 16 (b) 4 *Ex 2* (a) $4x - 6$ (b) $2x - 1$ *Ex 3* (a) $x \geq -1$ (b) $x \geq 1/2$ (c) $x \geq 1/2$

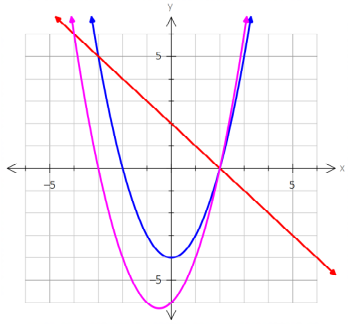
Practice Questions

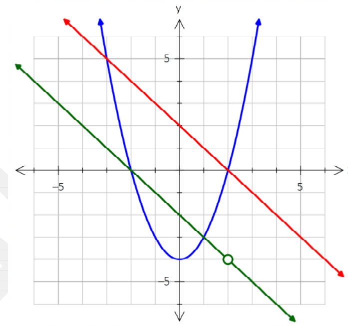
Function Operations and Transformations Question 1

Given functions $f(x) = x^2 - 4$ and $g(x) = 2 - x$, determine each function. (and label each graph)

(a) $h(x) = f(x) - g(x)$

(b) $k(x) = \frac{f(x)}{g(x)}$ *Also state domain*





(c) $p(x) = f \circ g(x)$

(d) $f(f(0))$

Function Operations and Transformations Question 2 (Extra Question)

Given functions $f(x) = \sqrt{3 - 2x}$ and $g(x) = 3x + 1$, determine each function, and state the domain.

(a) $\frac{f}{g}(x)$ (and state domain)

(b) $f \circ g(x)$

1.2 Function Transformations

Intro

$f(x)$ can be transformed to $y = af[b(x-h)]+k$

Fill out the table:

Equation in terms of f	Transformations (in words)	Mapping Notation	Replacements
	A vertical stretch by a factor of ¼ about the x-axis A reflection in the y-axis A horizontal translation 3 units left		
$2y-6=f(-\frac{1}{2}x+8)$			
		$(x,y)\to(3x+6,-y+2)$	
			$x\to 4x$ $y\to -y$ $x\to x-1$ $y\to y-3$

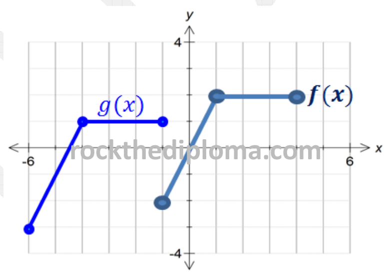
i – Horizontal and Vertical Translations

Translations involve adding / subtracting parameters.

$$y=f(x)\ \rightarrow\ y=f(x-\boldsymbol{h})+\boldsymbol{k}$$

Example 1:

State the equation of $g(x)$, in terms of $f(x)$.



Example 2:

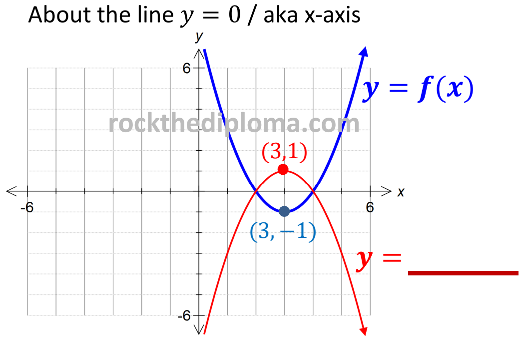
Describe the horizontal and vertical translations as $y=\sqrt{x}$ is transformed to $y=\sqrt{x+5}+1$

ii – Reflections

In this course we consider three types of reflections:

Vertical Reflections

About the line $y = 0$ / aka x-axis

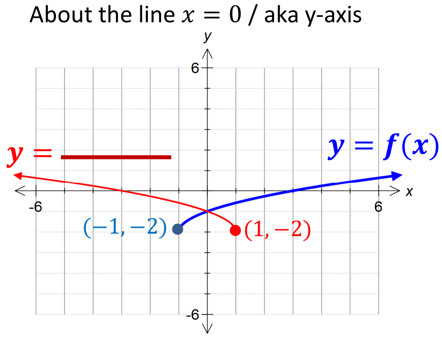


Mapping Rule: ____

Invariant Point: ____

Horizontal Reflections

About the line $x = 0$ / aka y-axis

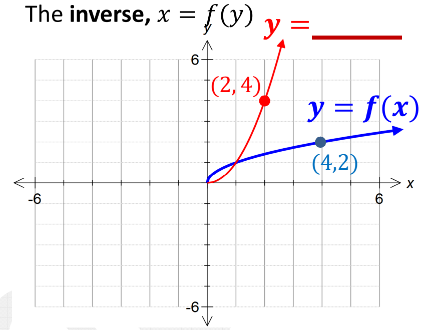


Mapping Rule: ____

Invariant Point: ____

Reflections About $y = x$

The **inverse**, $x = f(y)$



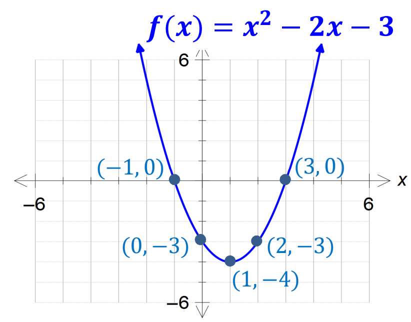
Mapping Rule: ____

Invariant Point: ____

Key Concept – Given a function in the form $y = f(x)$ with a horizontal or vertical reflection applied, we can obtain the equation and graph.

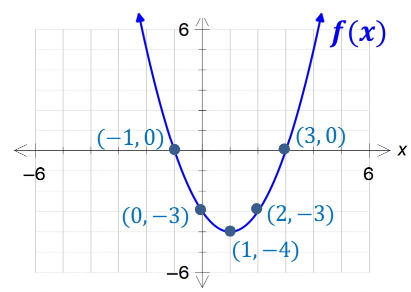
<u>**Vertical Reflection**</u> *(about line $y = 0$ / x-axis)* ☞ Replace "y" with "$-y$" *and simplify*

Example 1: Determine the **equation** and **graph** of the function $f(x) = x^2 - 2x - 3$ after reflection about the x-axis.



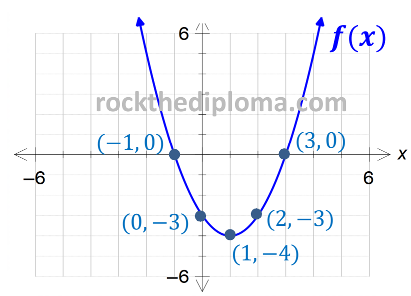
<u>**Horizontal Reflection**</u> *(about $x = 0$ / y-axis)* ☞ Replace "x" with "$-x$" *and simplify*

Example 2: Determine the **equation** and **graph** of the function $f(x) = x^2 - 2x - 3$ after reflection about the y-axis.



<u>**Inverse**</u> *(reflection about the line $y = x$)* ☞ Interchange "x" and "y"

Example 3: Determine the **graph** of the function $f(x) = x^2 - 2x - 3$ after reflection about the line $y = x$



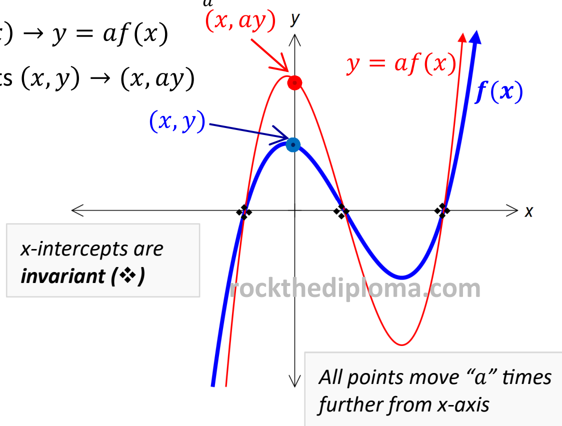
iii – Stretches

VERTICAL STRETCHING

In general, for a function $y = f(x)$ a **vertical stretch** occurs when a function is multiplied by some value, "a".
(Or when y is replaced with $\frac{1}{a}y$)

$y = f(x) \rightarrow y = af(x)$

All points $(x, y) \rightarrow (x, ay)$



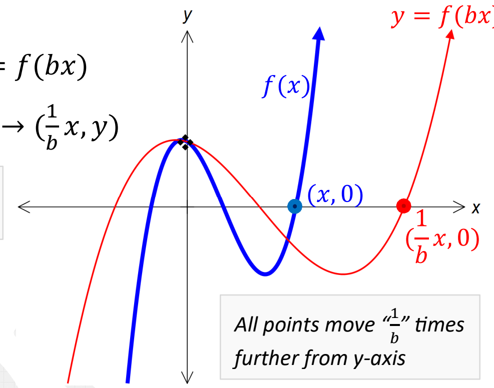
HORIZONTAL STRETCHING

In general, for a function $y = f(x)$ a **horizontal stretch factor of $\frac{1}{b}$** occurs when "x" is replaced with "bx" in the equation.

$y = f(x) \rightarrow y = f(bx)$

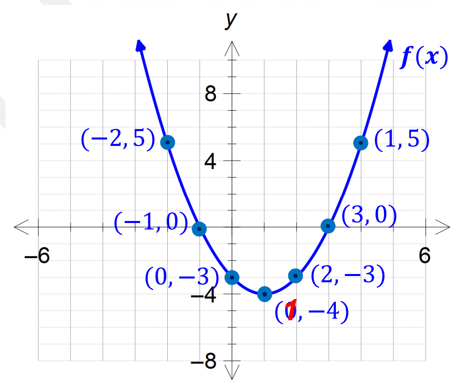
All points $(x, y) \rightarrow (\frac{1}{b}x, y)$

y-intercept is ***invariant*** ❖



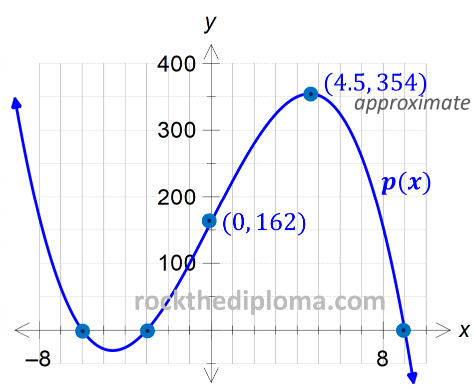
Example 4:

Given a function $f(x) = (x - 1)^2 - 4$, determine both the **equation** and the **graph** after a vertical stretch by a factor of 2.



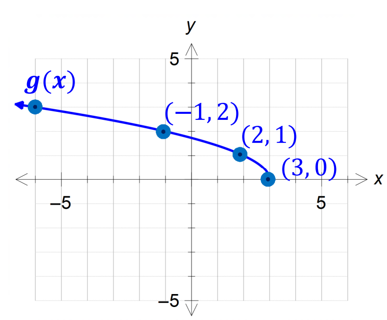
Example 5:

Given a function $p(x) = -(x + 6)(x + 3)(x - 9)$, determine both the **equation** (unsimplified) and the **graph** after a horizontal stretch by a factor of 1/3.



Example 6:

Given a function $g(x) = \sqrt{3 - x}$, determine both the **equation** and the **graph** after a horizontal stretch by a factor of 2.



Note: Vertical Shifts / Stretches are "straightforward"

$y=\sqrt{x}\ \rightarrow\ y=\sqrt{x}+7$ Shift 7 units up

$y=\sqrt{x}\ \rightarrow y=4\sqrt{x}$ Vertical stretch, *factor of 4*

Horizontal Shifts / Stretches are trickier

Opp sign ➔ $y=\sqrt{x}\ \rightarrow\ y=\sqrt{x+7}$ Shift 7 units left

Reciprocal ➔ $y=\sqrt{x}\ \rightarrow y=\sqrt{4x}$ Horizontal stretch, *factor of* $1/4$

When applying two or more transformations to a function in the form $y=af[b(x-h)]+k$, we apply the stretches / reflections FIRST, then apply any horizontal or vertical translations.

The order of transformations can be abbreviated **SRT**.

The mapping rule for $y=f(x)\ \rightarrow\ \ y=af[b(x-h)]+k$ is:

All points $(x,y)\rightarrow(\frac{1}{b}x+h,ay+k)$ *Note: If a or b are* <0*, then a reflection also occurs*

For example, the mapping rule for a function $y=-3g[2(x+1)]-4$ is:

All points $(x,y)\rightarrow(\frac{1}{2}x-1,-3y-4)$

Vertical Reflection about the line $y=0$
Vertical stretch by a factor of 3 ➔ *...then shift 4 units down*

Horizontal stretch by a factor of $1/2$ ➔ *...then shift 1 unit left*

Practice Questions

Function Operations and Transformations Question 5

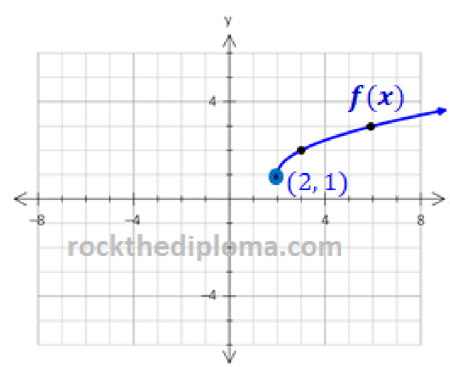
Describe the transformations that occur from $y=f(x)$ to $y=-f(x+3)-1$. Then, determine the coordinates of a point on $f(x)$, $P(3,9)$ using a *mapping rule*.

Function Operations and Transformations Question 6

A function $f(x)$, as shown, is reflected about the y axis, and vertically translated four units down.

a) State the domain and range of the resulting function.

b) State the equation of the resulting function, in terms of $f(x)$



3.3 Laws of Logarithms

Laws of Logarithms

❶ $\log_b(M \times N) = \log_b M + \log_b N$

❷ $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

❸ $\log_b(M^n) = n \log_b M$

$\log_b c = \dfrac{\log_a c}{\log_a b}$ ← ***Change of base*** *identity*

☞ The **first two** log laws are typically used to either:

- *Write two or more log terms as a single log*

$$\boldsymbol{log 20 + log 50} = log(20 * 50) = log(1000) = \mathbf{3}$$

or
- *Split up a log term into two or more logs*

$$\boldsymbol{log_2\left(\frac{x}{8}\right)} = log_2 x - log_2 8 = \boldsymbol{log_2 x - 3}$$

☞ The **third log law** is often used
- As a first step in writing expressions as a single log:

$\boldsymbol{2logx - 3logy}$

$= logx^2 - logy^3$

$= log\dfrac{x^2}{y^3}$

Or, • To evaluate log expressions:

If $\boldsymbol{log_5 x = 7}$, evaluate $\boldsymbol{log_5 x^3}$

$log_5 x^3 = 3log_5 x$ (*Given – this is "7"*)

$= 3(7)$

$= \mathbf{21}$

The *Change of Base* Identity

Laws of Logarithms

$\log_b(M \times N) = \log_b M + \log_b N$

$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

$\log_b(M^n) = n \log_b M$

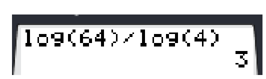
$\log_b c = \dfrac{\log_a c}{\log_a b}$

Says: *Any log expression can be written using any other base!*

Example: *base!* $log_4 64$ *can be written* $\dfrac{log_2 64}{log_2 4}$ ← 6, ← 2

This is "3" — **Also "3"**

Even better *than re-writing* $log_4 64$ *in base 2 is using the calculator default – base 10:*



Practice Questions

➡ ***Exp. Functions and Logs Question 18:***

Write as a Single Log and Then Evaluate:	(a) Evaluate by writing as a single log: $log 40 + log 500 - log 2$
❖ $\boldsymbol{log_3 6 - log_3 4 + log_3 18}$ $= log_3 \dfrac{6 \times 18}{4}$ $= log_3 27$ ➢ $= \mathbf{3}$	Ans: 4

Write as a Single Log and Then Evaluate:

❖ $\frac{1}{2}\boldsymbol{log_5 175} - \frac{1}{2}\boldsymbol{log_5 7}$

$= log_5(175)^{\frac{1}{2}} - log_5(7)^{\frac{1}{2}}$

$= log_5 \frac{(175)^{\frac{1}{2}}}{(7)^{\frac{1}{2}}}$ ➔ $= log_5\left(\frac{175}{7}\right)^{\frac{1}{2}}$

$= log_5(25)^{\frac{1}{2}}$

$= \frac{1}{2}log_5 25$ ➢ $= \frac{1}{2}(2)$ ➢ $= \mathbf{1}$

(b) Evaluate by writing as a single log: $\frac{1}{2}log_3 144 - log_3 4 + 2log_3 3$

Ans: 3

Write as a Single Log:

❖ $\boldsymbol{3log_3 a + 2log_3 b - (\frac{1}{2}log_3 c + log_3 a)}$

$= log_3 a^3 + log_3 b^2 - log_3 c^{\frac{1}{2}} - log_3 a$

There are four log terms here, *Positive log terms, ship the "a^3" and "b^2" to the top. (Negative log terms, the bottom)*

$= log_3 \frac{a^3 b^2}{c^{1/2} a}$

$= \boldsymbol{log_3 \frac{a^2 b^2}{\sqrt{c}}}$

(c) Write as a single log: $2logA + \frac{1}{3}logC - (2logC - 3logB)$

Ans: $log\frac{A^2B^3}{C^{5/3}}$

❖ If $\log 17 = k$ determine and expression for each of the following:

(a) $\boldsymbol{log170}$

$= \log(17 * 10)$

$= log17 + log10$

$= k + 1$

(b) $\boldsymbol{log1.7^2}$

$= 2log1.7$

$= 2log\frac{17}{10}$

$= 2(log17 - log10)$

$= 2(k - 1)$

(d) If $\log 8 = m$ determine an expression for each of the following:

(i) $log800$ (ii) $log\sqrt{512}$

(i) $m + 2$ (ii) $\frac{3}{2}m$

❖ **If $\boldsymbol{log_3 4 = x}$, express $\boldsymbol{log_3 64}$ in terms of x:**

$log_3 64 = log_3(4^3)$

$= 3log_3 4$

$= 3x$

(e) If $\log_2 9 = x$, express each of the following in terms of "x":

(i) $log_2 162$ (ii) $log_2\frac{729}{\sqrt{2}}$

$(i)\ 2x + 1$ $(ii) 3x - \frac{1}{2}$

❖ **If $\boldsymbol{log_3 x = 8}$, evaluate $\boldsymbol{log_3(9x)}$**

$log_3(9x) = log_3 9 + log_3 x$

$= 2 + 8$

$= 10$

(f) If $\log_5 x = 2$, evaluate each of the following:

(i) $\log_5 5x^3$ (ii) $\log_5 \frac{x^2}{25}$

(i) 7 (ii) 2

3.4 Exponential and Logarithmic Equations

i – Solving any Exponential Equation

Using logarithms provides a method to solve **any exponential equation**.

➔ Some exponential equations can be solved by equating the bases
Provided re-writing in the same base is possible…

Example: $2^{x-1} = \frac{1}{8}$ ➔ $2^{x-1} = 2^{-3}$
$x - 1 = -3$ $\quad \boldsymbol{x = -2}$

➔ ALL exponential equations can be solved by "logging both sides" / using the log law $log_b M^n = nlog_b M$

***Example:* Solve** $\mathbf{3(2)^{x-1} = 120}$ *(nearest hundredth)*

Solve by *LOGGING BOTH SIDES*

❶ Isolate power term
(Divide both sides by "3") $\qquad (2)^{x-1} = 40$

❷ *LOG BOTH SIDES (base 10)* $\qquad log(2)^{x-1} = log40$

❸ Use log law:
$log_b M^n = nlog_b M$ $\qquad (x-1)log2 = log40$

$$x - 1 = \frac{log40}{log2}$$

❹ Now that "x" is out of the exponent, isolate

$$\boldsymbol{x \approx 6.32}$$

Solve by *CONVERTING TO LOG FORM*

❶ Isolate power term
(Divide both sides by "3") $\qquad (2)^{x-1} = 40$

❷ *CONVERT TO LOG FORM* $\qquad log_2(40) = x - 1$

❸ Use change of base:
$log_b c = \frac{log_a c}{log_a b}$

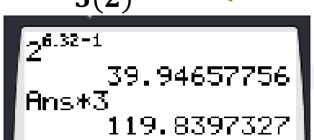
$$\frac{log(40)}{log(2)} = x - 1$$

$$x = \frac{log40}{log2} + 1$$

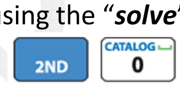
$$\boldsymbol{x \approx 6.32}$$

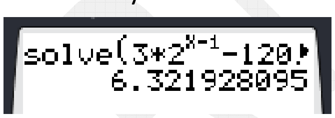
Note that this equation can be ***verified numerically***:

$\mathbf{3(2)^{6.32-1}}$ ← Substitute $x = 6.32$ into the left side of the equation, works out to *close to 120. (the right side)*



… Or for the especially bold by using the "***solve***" feature of your calculator





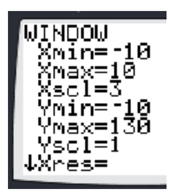
$\color{darkred}{\textbf{solve}(3 * 2^{x-1}\ ,\ x\ ,10)}$

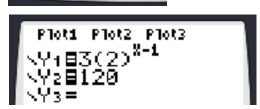
Equation must be set to "0", input the Left Side

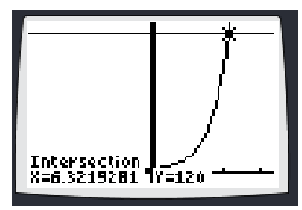
Solving for x

This number is a "guess", only relevant if there is more than 1 solutions

… Or ***graphically***:







Graph **either** y_1 = *left side* , y_2= *right side* and get the point of intersection
(Although you don't need to actually see the point of intersection, you'll have to make the y-max at least 120 if you want to!)

OR, first set the equation to "0", graph $y_1 = 3 * 2^{(x-1)} - 120$ and get the zero

Practice Questions

Exp. Functions and Logs Question 28:

<table>
<tr><th>Worked Example...</th><th>You Try... (State answers as exact values, and to the nearest hundredth)</th></tr>
<tr><td>$3(2)^{x-1} = 120$
$(2)^{x-1} = 40$
$log(2)^{x-1} = log40$
$(x-1)log2 = log40$
$x - 1 = \frac{log40}{log2}$ ➡ $x \approx 6.32$</td><td>➡ (a) $\frac{1}{5}(1.2)^{2x+1} = 3$

6.93</td></tr>
<tr><td>$8^{2x} = 37^{x-4}$
$log\ 8^{2x} = log37^{x-4}$
$2xlog8 = (x-4)log37$
$2xlog8 = xlog37 - 4log37$
Group the terms with "x" on the same side, then factor out
$4log37 = xlog37 - 2xlog8$
$4log37 = x(log37 - 2log8)$
$\frac{4log37}{log37 - 2log8} = x$ ➡ $x \approx -26.36$</td><td>(b) $2^{x+3} = 17^{x}$ (Extra Question)

0.97</td></tr>
<tr><td>The half-life of a particular substance is 45.2 years. If there was initially 4.6g of the substance, find how long it would take for the substance to decay to 1g. (Nearest tenth)
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Use $y = ab^{\frac{t}{p}}$
Think? Are you given "p"? (the doubling period / half-life), or do you want p?
$1g = 4.6g(\frac{1}{2})^{\frac{t}{45.2}}$
➔ Here we are GIVEN p! (the half life is 45.2 years)
$\frac{1}{4.6} = (\frac{1}{2})^{\frac{t}{45.2}}$
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$\log\left(\frac{1}{4.6}\right) = \log\left[(\frac{1}{2})^{\frac{t}{45.2}}\right]$
$log\left(\frac{1}{4.6}\right) = \frac{t}{45.2}log\left(\frac{1}{2}\right)$
$t = \frac{45.2\log(1/4.6)}{\log(1/2)}$ $t \approx 99.5$ yrs</td><td>➡ (c) A particular substance is decaying exponentially so that only 10% of the original amount remains after 22.4 hours. Find the half-life of the substance. (Nearest tenth)

6.7 hrs</td></tr>
<tr><td>An investment of $5000 is compounded annually at a rate of 3.25%. Algebraically determine the length of time it would take for the value of the investment to reach $10000. (Nearest tenth)
Use $y = ab^{t}$
$10\ 000 = 5\ 000(1.0325)^{t}$
$\frac{10\ 000}{5\ 000} = (1.0325)^{t}$
$\mathbf{log}(2) = \mathbf{log}1.0325^{t}$
$\mathbf{log}(2) = t\mathbf{log}1.0325$
$t = \frac{log2}{log1.0325}$ $t \approx 21.7$ yrs</td><td>➡ (d) A particular city has a population of 25 400 and is decreasing exponentially at a rate of 1.6% per year. Algebraically determine the amount of time it would take for the population to reach 20 000. (Nearest tenth)

14.8 yrs</td></tr>
</table>

ii – Logarithmic Equations

There are two types of ***logarithmic equations*** we need to be concerned with, with two methods to solve:

Simplify with log laws (if necessary) **and then** ***drop the logs***

$$\boldsymbol{log_2 x + log_2(x-2) = log_2 3}$$

Combine term on left side using the first log law $\quad log_2[x(x-2)] = log_2 3$

Drop the logs! $\quad x^2 - 2x = 3$

$x^2 - 2x - 3 = 0$

Factor to solve $\quad (x-3)(x+1) = 0$

$\boldsymbol{x = 3}$ ***or*** ~~$\boldsymbol{x = -1}$~~

$x = -1$ is ***extraneous*** *as we must "throw out" any solutions which would have us logging negatives*

Simplifying by log laws (if necessary) **and then** ***converting to exponential form***

$$\boldsymbol{log_2 x + log_2(x-2) = 3}$$

Convert to ***exponential form*** $\quad log_2[x(x-2)] = 3$

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$2^3 = x^2 - 2x$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

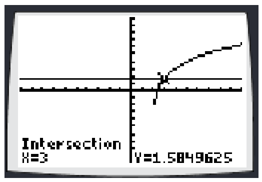
$\boldsymbol{x = 4}$ ***or*** ~~$\boldsymbol{x = -2}$~~

$x = -2$ is ***extraneous***

Solve graphically!

$y_1 = log(x) \div log(2) + log(x-2) \div log(2)$

$y_2 = log(3) \div log(2)$ ➔



Exp. Functions and Logs Question 29: Use an algebraic process to solve each equation for x.

(a) $log_3(x+1) + 3log_3 2 = \frac{1}{2}log_3 144$

(b) $log_5 x - log_5(x-2) = 3$

Exp. Functions and Logs Question 30: *(Extra Question)*

(b) $log_2(x-5) + log_2(x-2) = 2$

(b) $log_5(x+5) - log_5(x+1) = log_5(3x)$

Exp. Functions and Logs Question 31

A student is asked to solve the equation $\dfrac{125^{x(x+1)}}{5^{(3x-4)}} = 25^{(x-5)}$ using an algebraic process.

She is able to simplify the equation to the form $3x^2 + bx + c = 0$.

Diploma Example

The value of c is

A. 6

B. 9

C. 14

D. 40

Question 2

Graph is shifted 4 units left and 1 unit down. (From graph of $= \sqrt{x}$)

So equation is $y = \sqrt{b(x+4)} - 1$. Use any point (other than the "starting point") to solve for b. I'll use the point (-2, 0)…

$\mathbf{0} = \sqrt{b(\mathbf{-2}+4)} - 1$

$1 = \sqrt{b(2)}$ square both sides

$1 = 2b$

$b = \mathbf{1/2}$

Answer:

$$y = \sqrt{\mathbf{0.5(x+4)}} - \mathbf{1}$$

**Make sure the "x+4" is in brackets. Can you see why?*

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Question 6

Function $y = \sqrt{16 - x^2}$ has a **domain** of $\{-4 \le x \le 4\}$. *(Defined where $f(x) \ge 0$; that is, where the graph of $f(x)$ is on or above the x-axis)*

The **range** is $\{0 \le y \le 4\}$

The **invariant points** are where $f(x) = 0$ ➔ At **(-4,0)** and **(4,0)**

…Or where $f(x) = 1$:

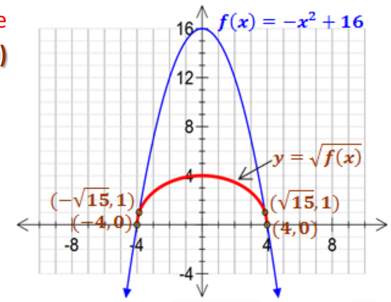
$-x^2 + 16 = 1$

$15 = x^2$

$x = \pm\sqrt{15}$

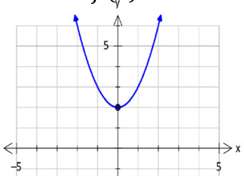
Points are:

$(-\sqrt{15}, 1)$ and $(\sqrt{15}, 1)$

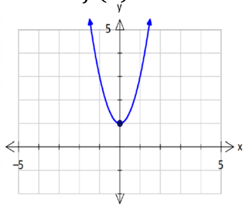


Question 8

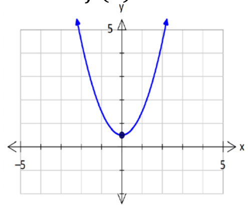
(a) $k > 1$ Such as $f(x) = x^2 + 2$



(b) $k = 1$ Such as $f(x) = 2x^2 + 1$



(c) $0 < k < 1$ Such as $f(x) = x^2 + 0.5$



(d) $k = 0$ Such as $f(x) = 3x^2$ (e) $k < 0$ Such as $f(x) = x^2 - 1$

Question 9

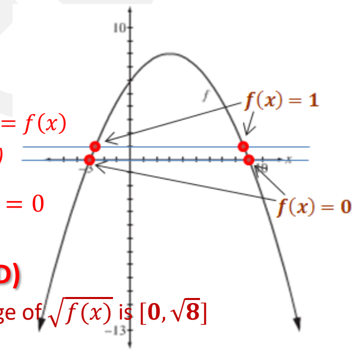
Invariant points occur where $f(x) = 0$ or $f(x) = 1$.

(Since we are transforming from $y = f(x)$ to $y = \sqrt{f(x)}$ and $\sqrt{0} = 0$, $\sqrt{1} = 1$)

So, draw to horizontal lines at $y = 0$ and $y = 1$, there are

FOUR invariant points. ANS: (D)

Range of $f(x)$ is $(-\infty, 8]$. So range of $\sqrt{f(x)}$ is $[0, \sqrt{8}]$



Question 12

Asymptotes are shifted 1 left, 2 down. So equation form is $y = \frac{a}{x+1} - 2$. Use any point on the graph, such as (0, 1), to solve for "a".

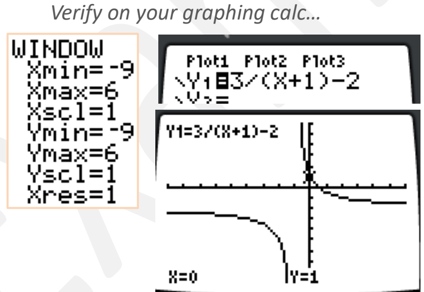
$\mathbf{1} = \frac{a}{\mathbf{0}+1} - 2$

$3 = \frac{a}{1}$

$a = 3$

$$y = \frac{\mathbf{3}}{\mathbf{x+1}} - \mathbf{2}$$

Verify on your graphing calc…



Note: This equation,

$y = \frac{3}{x+1} - 2$

Can be ***simplified***…

$y = \frac{3}{x+1} - \frac{2(x+1)}{x+1}$

$y = \frac{3 - 2x - 2}{x+1}$ ➔ $y = \frac{-2x+1}{x+1}$

Notice that the ratio of the lead coefficients (horz. asymp.) Is "-2"!

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Question 15

(a) $y = \frac{\cancel{x+3}}{(x-5)\cancel{(x+3)}}$ ➔ $y = \frac{1}{(x-5)}$; $x \ne -3$ or 5

Domain: $\{x \ne -3 \text{ or } 5\}$ **Range:** $\{y \ne 0 \text{ or } -\frac{1}{8}\}$

Discontinuities: VA at $x = 5$ (factor doesn't cancel)

PD at $x = -3$ (factor cancels)

y-intercept: $-\frac{1}{5}$ **x-intercept:** *none*

Set $x = 0$ in $y = \frac{1}{x-5}$

For y-coord of PD, substitute -3 into $y = \frac{1}{x-5}$

(b) $y = \frac{x\cancel{(x-4)}}{(x-1)\cancel{(x-4)}}$ ➔ $y = \frac{x}{x-1}$; $x \ne 1$ or 4

H.A. at $y = 1$…same degree top and bottom so ratio of lead coefficients! ($\frac{1x}{1x-1}$)

Domain: $\{x \ne 1 \text{ or } 4\}$ **Range:** $\{y \ne 1 \text{ or } \frac{4}{3}\}$

Discontinuities: VA at $x = 1$ (factor doesn't cancel)

PD at $x = 4$ (factor cancels)

y-intercept: 0 **x-intercept:** 0

Set $x = 0$ in $y = \frac{x}{x-1}$

For y-coord of PD, substitute 4 into $y = \frac{x}{x-1}$

Question 16 (b)

$$y = \frac{(2x+1)(x-3)}{x(x-3)}$$

x-intercept at 1/2 *(factor must be on top only)*

V.A. at $x = 0$ *(factor must be on bottom only – doesn't cancel)*

P.D. at $x = 3$ *(factor must cancel out)*

ALSO NOTE:

The graph has a H.A. at y = 2, which means that the **ratio of the lead coefficients** must be 2.

Which we do have here > "$2x^2$" on top (when expanded out), and "x^2" on the bottom. (2/1 = 2)